

Logic Programming Engineering – Assignment 6

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Exercise 6.1 [Scalar product]

Define a predicate `scalar/3` which for two given vectors $\vec{v}_1 = (a_1, a_2, \dots, a_n)$ and $\vec{v}_2 = (b_1, b_2, \dots, b_n)$ of integers (i.e., $a_i, b_i \in \mathbb{Z}$, $i = 1, \dots, n$) computes the scalar product. Please remind: The scalar product (aka dot product) of $\vec{v}_1$ and $\vec{v}_2$ is defined as $\vec{v}_1 \cdot \vec{v}_2 = \sum_{i=1}^n a_i b_i$.

Example:

```
?- scalar([3, 2, 5], [5, 3, 6], S).  
S = 51.
```

Exercise 6.2 [Polynomials]

Polynomials can be represented as lists of pairs of coefficients and exponents. For example the polynomial $4x^5 + 2x^3 - x + 27$ can be represented as the following Prolog list: `[(4,5), (2,3), (-1,1), (27,0)]`.

- a) Provide a predicate `polyValue/3` that computes the polynomial for a given value of the variable $x \in \mathbb{R}$.

Examples:

```
?- polyValue(7, [(4,5), (2,3), (-1,1), (27,0)], R).  
R = 67934.
```

```
?- polyValue(5.5, [(4,5), (2,3), (-1,1), (27,0)], R).  
R = 20485.625.
```

- b) Now provide an alternative predicate called `horner/3` that does the same as `polyValue/3`, but follows the Horner scheme to compute the polynomial for a given value of the variable $x \in \mathbb{R}$, e.g.,

$$3x^3 + 2x^2 - 5x + 4 = 4 + x(-5 + x(2 + x(3)))$$

For this task you may, e.g., assume that the polynomial is given as a complete and sorted list whose order is increasing (or decreasing) in the exponents of the polynomial, i.e., the polynomial $4x^5 + 2x^3 - x + 27$ would be given as the Prolog list `[(27,0), (-1,1), (0,2), (2,3), (0,4), (4,5)]`.

- c) Create a random generator for polynomials with a given maximum degree and maximum coefficient size. For this, provide a predicate `randomPoly/3`. The format of the list should be compatible with the parts a) and b) of this exercise.

Examples:

```
?- randomPoly(P,5,20).  
P = [(9, 0), (12, 1), (14, 2), (6, 3), (19, 4), (19, 5)].
```

```
?- randomPoly(P,5,20).
```

```
P = [ (18, 0), (7, 1), (12, 2), (10, 3), (5, 4), (19, 5)].
```

- d) Provide a predicate `polyTester` using `randomPoly/3` from part c) to create random polynomials and compute the polynomial for random values of $x \in \mathbb{N}$ with the help of `polyValue/3` and `horner/3`. Use the new predicate for comparing the two methods with respect to their efficiency. Hint: you may for example use the predicate `time/1` or the SWI-Prolog profiler (cf. `profile/1`).
- e) Write a predicate `polysum/3` for adding two polynomials. Your solution should work independent of the ordering of pairs inside the two given lists.

```
?- polysum([(2,2), (3,1), (5,0)], [(5,3), (1,1), (10,0)], X).
```

```
X = [(4,1), (15,0), (2,2), (5,3)].
```